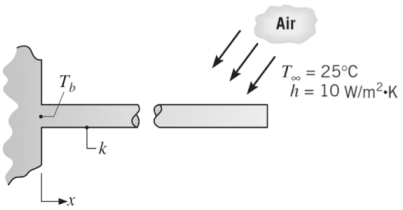
Name: Jason Kirkland

Submitted: 03/10/2026

Problem set #5

Due: 03/10/2026

1. (60 pt) A cylindrical pin fin with a radius of $r = 1$ mm and length of $L = 5$ cm is exposed to airflow with a temperature of $T_\infty = 25$ °C and a heat transfer coefficient of $h = 10$ W/m^2·K. Thermal conductivity of the pin fin is $k = 200$ W/m·K. The tip of the pin fin is thermally insulated. Use the fin problem solving workflow discussed in the lecture to solve the following questions.



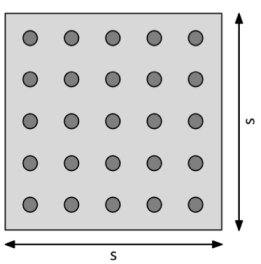
(a) If the fin base temperature is $T_b = 125$ °C, please calculate the fin temperature at $x = L/2$ and $x = L$.

(b) Calculate the fin effectiveness ε_f and the fin efficiency η_f based on their definition and results provided by Table 3.4 of the textbook.

(c) Calculate the fin efficiency η_f based on results provided by Table 3.5. Compare the results of (b) and (c) and discuss if Table 3.5 provided a good approximation to the fin efficiency.

(d) Calculate the thermal resistance of this pin fin R_f.

(e) The pin fins then form a 5×5 array mounted on a base plate with a side length of $s = 3$ cm, as shown in the figure on the right-hand side. Calculate the overall efficiency of the fin array η_o and the thermal resistance of the fin array R_o. Calculate how much heat is dissipated through the unfined surface and how much is through the fins.



Initial knowns:

- Cylindrical Pin fin

$r = 0.001$ m
$L = 0.05$ m
$T_\infty = T_{air} = 25$ °C
$h = 10$ W/m^2·K
$k = 200$ W/m·K

Find:

a) Temp @ $x = L$

b) ε_f & η_f

c) η_f

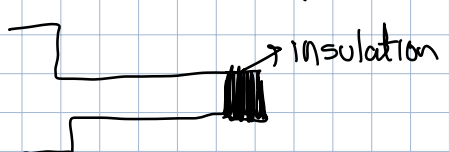
d) R_f thermal resistance

e) Overall values

Properties: • fin mechanics

Assume: • Ideal fin conditions

Analysis:



a) $T_b = 125$°C – fin temp @ $x = \frac{L}{2}$

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

$$-k\left.\frac{dT}{dx}\right|_{x=L} = 0$$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh(mL)}$$

$$= \frac{\cosh\left(0.5\cdot\left(0.05-\frac{0.05}{2}\right)\right)}{\cosh(10)}$$

$$\frac{T\left(\frac{L}{2}\right)-T_\infty}{\theta_b} = 0.91467$$

$$m = \sqrt{\frac{hP}{kA_c}}$$

$$= \sqrt{\frac{(10)2\pi(0.001)}{(200)\pi(0.001)^2}}$$

$m = 10\ m^{-1}$

$mL = 10(0.05) = 0.5$

$\theta_b = T_b - T_\infty$

$= 125 - 25$

$\theta_b = 100$

$$T\left(\tfrac{L}{2}\right) - 25 = 100(0.914)$$

$$\boxed{T\left(\tfrac{L}{2}\right) = 116.467^\circ C}$$

$$\frac{\theta}{\theta_b} = \frac{\cosh(0)}{\cosh(10)}$$

$$\frac{T(L) - T_\infty}{100} = 0.886$$

$$T(L) - 25 = 88.68$$

$$\boxed{T(L) = 113.68^\circ C}$$

b) $\varepsilon_f = \dfrac{q_f}{q_{nf}} > 1$ $\qquad q_f = M\tanh(mL)$

$q_f = 0.628\tanh(0.5)$

$= 0.290$

$M = \sqrt{hPkA_c\theta_b}$

$= \sqrt{10(2\pi(0.001))\,200\pi(0.001)^2(125-25)}$

$M = 0.628$

$m = 10\ m^{-1}$

$mL = 0.5$

$\eta_f = \dfrac{q_f}{hA_f\theta_b} = \dfrac{0.29}{(10)(2\pi(0.001)0.05)100}$

$= hA(T_f - T_\infty)$ (conv)

$\boxed{\eta_f = 0.924}$

$\varepsilon_f = \dfrac{q_f}{hA_c\theta_b} = \dfrac{0.29}{(10(\pi(0.001)^2)100} = \boxed{\varepsilon_f = 92.42}$

c) $L_c = L + \dfrac{A_c}{P}$

$= 0.05 + \dfrac{\pi(0.001)^2}{2\pi(0.001)}$

$= 0.0505$

$\eta_f = \dfrac{\tanh mL_c}{mL_c}$

$\eta_f = \dfrac{\tanh(10\cdot 0.0505)}{(10\cdot 0.0505)} = \boxed{0.923}$

$\eta_{fb} \approx \eta_{fc}$

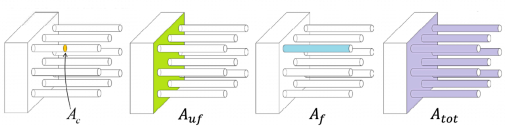
$\boxed{\eta_{fb} - \eta_{fc} = 0.00138}$

d) $R_f = ?$

$$R_f = \frac{T_b - T_\infty}{q_f} = \frac{100}{0.29} = \boxed{R_f = 344.404}$$

e) $A_{tot} = NA_f + A_{uf}$ (through fins + unfinned)

Surface areas



$$A_{tot} = NA_f + A_{uf}$$

Comment:
1. Generally, the tip area can be neglected from A_f and A_{tot} because it is small compared to surface area.

$$NA_f = 25(L \cdot 2\pi r)$$

$$= 25(0.05)(2\pi(0.001))$$

$$NA_f = 0.00785$$

$$A_{uf} = A_{base} - N(A_c)$$

$$= (0.03)^2 - 25(\pi(0.001)^2)$$

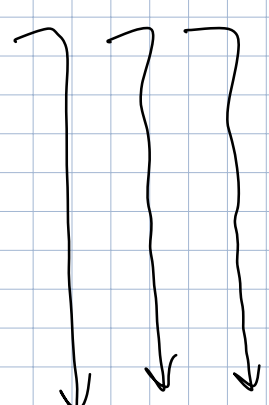
$$A_{uf} = 8.21E\text{-}4\ m^2$$

$$\eta_o = \frac{N\eta_f A_f + A_{uf}}{A_{tot}} = \frac{(25)(0.924)(3.14E\text{-}4) + 8.21E\text{-}4}{8.21E\text{-}4 + 0.00785}$$

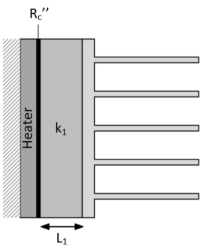
$$\boxed{\eta_o = 0.9311}$$

$$R_o = \frac{1}{\eta_o h A_{tot}} = \frac{1}{(0.9311)(10)(0.00836)} = \boxed{R_o = 12.379}$$

f)



(f) The heat sink consisting of this fin array is used to improve the heat dissipation from a thin film heater, as shown in the figure on the right-hand side. The heat sink is mounted on a slab with thickness L_1 = 2 cm and thermal conductivity k_1 = 10 W/m·K. There is a thermal adhesive between the thin film heater and the slab, resulting in a contact thermal resistance R_c = 10^{-4} m²·K/W. Thermal insulation was applied to the backside of the heater. The heater generates a uniform heat flux q'' = 1000 W/m², which was dissipated through the heat sink. The temperature across the thin film heater is uniform. Thermal resistance for the base of the heat sink can be neglected. There is no contact thermal resistance between the base of the heat sink and the slab. Please draw the thermal resistance network of this system and calculate the temperature of the thin film heater T_h. Compared with the condition without the heat sink, *i.e.*, directly exposing the heater and the slab to ambient air, how much temperature reduction of the heater is achieved?



W DC

$L_1 = 0.02\,m$

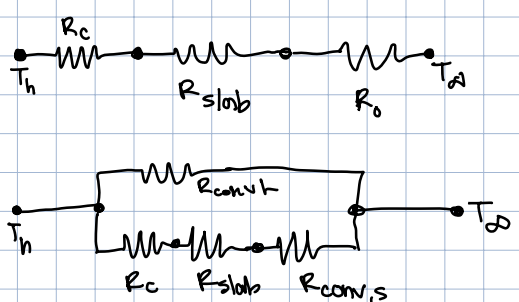
$k_1 = 10\ \frac{W}{m\cdot k}$

$R_c = 10^{-4}\ \frac{m^2 k}{W}$

$q'' = 1000\ \frac{W}{m^2}$

$T_h = ?$

$T_h \rightarrow T_{A_2} \xrightarrow{R_c} T_s \rightarrow R_0 \rightarrow T_\infty$



w/ HS:

$R_c = \frac{R_c}{A}$

$R_{slab} = \frac{L_1}{k_1 A}$

$R_0 = 12.4\ \frac{K}{W}$

$R_{tot} = \frac{R_c}{A} + \frac{L_1}{k_1 A} + 12.4$

$A = (0.03)^2 = 9E\text{-}4$

$= \frac{10E\text{-}4}{9E\text{-}4} + \frac{0.02}{(10)(9E\text{-}4)} + 12.4 = 14.73\ K/W$

$T_h - T_\infty = q R_{tot}$

$T_h - 25 = 0.9(14.73)$

$T_h = 38.3°$

$q_t = q'' A$

$= 1000(0.03)^2$

$q_{tot} = 0.9$

w/o HS:

$$R_s = \frac{1}{hA} = \frac{1}{(10)(9E\text{-}4)} = 111.11 \ \frac{k}{w}$$

$$\frac{1}{R_{tot}} = \frac{1}{hA} + \frac{1}{R_{slab} + R_{conv} + R_c}$$

$$\frac{1}{R_{tot}} = \frac{1}{111.11} + \frac{1}{113.44}$$

$$R_{tot} = 56.1 \ k/w$$

$$T_h - T_\infty = q'' R$$

$$T_h - 25 = (0.9)(56.1)$$

$$\boxed{T_h = 75.5^\circ C}$$

$$\boxed{T_{h2} - T_{h1} = 37.2^\circ C}$$

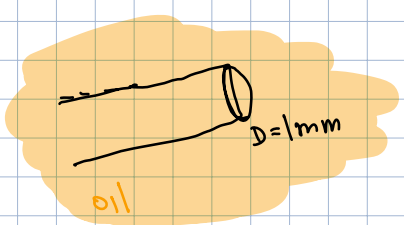
2. (30 pt) A long wire of diameter $D = 1$ mm is submerged into an oil bath of temperature $T_\infty = 20$ °C. Density ρ, thermal conductivity k, and heat capacity c of the wire are 4000 kg/m^3, 400 W/m·K, and 500 J/kg·K, respectively. The wire has an electrical resistance per unit length of $R_e' = 0.01$ Ω/m. The oil bath can dissipate heat with a heat transfer coefficient of $h = 500$ W/m^2·K. At the initial moment $t = 0$ s, a current of $I = 100$ A flows through the wire and produces Joule heating.

(a) Calculate the steady-state temperature of the wire.

(b) Apply the lumped capacitance method to derive an expression for the wire temperature as a function of time. How long does it take for the wire to reach a temperature that is within 5 °C of the steady-state temperature?

Given:
$D = 0.001$ m
$T_\infty = 20°C$
$\rho = 4000$ kg/m^3
$k = 400$ w/m·k
$c = 500$ J/kg·k

$R_e' = 0.01$ Ω/m
$h = 500$ w/m^2·k
$t_0 = I$
$I = 100$



Find: · SS temp
· lumped capacitance

Properties: convection & conduction

Assume: Joule heating

Analysis:

A) $\dot{q}_g = I^2 R$

$= 100^2 (0.01)$

$\dot{q}_g = 100$ w/m

$V = \frac{\pi D^2}{4} = 7.85E\text{-}7$ $\quad A_s = \pi D = 0.003141$

$\dot{q} = hA(T_s - T_\infty)$

$100 = 500(\pi \cdot 0.001)(T_s - 20)$

$100 = 1.57(T_s - 20)$

$63.66 = T_s - 20$

$83.66°C = T_s$

b) $\tau = \frac{\rho V C}{h A_s} \rightarrow \tau = 1 \text{ second}$

$\dot{q}_g - hA_s(T - T_\infty) = \rho V C \frac{dT}{dt}$

$\frac{-hA_s}{\rho V C}(T - T_\infty) = \frac{dT}{dt}$

$\frac{dT}{dt} + \frac{hA_s}{\rho V C}(T - T_\infty) = \frac{\dot{q}_g}{\rho V C}$

$T(t) = T_{ss} + (20 - T_{ss})e^{-t/\tau}$

$T(t) = 83.66 + (20 - 83.66)e^{-t}$

$T(t) = 83.66 - 63.66e^{-t}$

$|T_{ss} - T(t)| = 5$

$5 = (83.66 - 20)e^{-t}$

$\boxed{t = 2.54}$

(Q3) - This assignment

Sibley School of Mechanical and Aerospace Engineering

MAE 3240 Heat Transfer

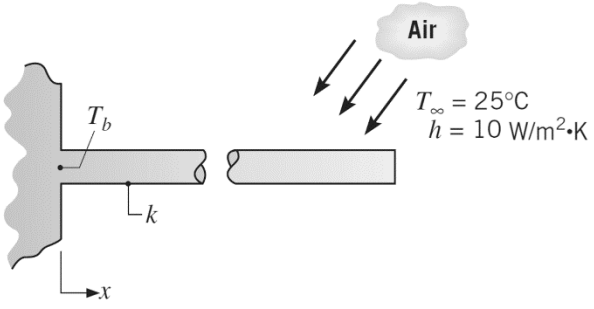
Spring 2026

Problem Set #5

Posted: Monday, Mar 2 **Due: Tuesday, Mar 10, 11:59 pm**

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1. (60 pt) A cylindrical pin fin with a radius of r = 1 mm and length of L = 5 cm is exposed to airflow with a temperature of T_∞ = 25 °C and a heat transfer coefficient of h = 10 W/m^2·K. Thermal conductivity of the pin fin is k = 200 W/m·K. The tip of the pin fin is thermally insulated. Use the fin problem solving workflow discussed in the lecture to solve the following questions.



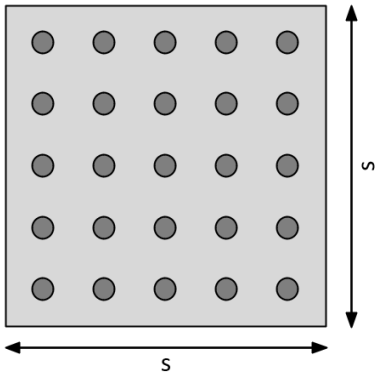
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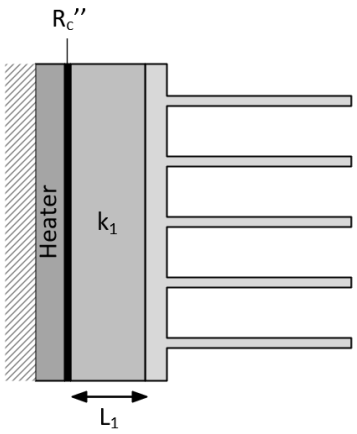
   (c) Calculate the fin efficiency η_f based on results provided by Table 3.5. Compare the results of (b) and (c) and discuss if Table 3.5 provided a good approximation to the fin efficiency.

   (d) Calculate the thermal resistance of this pin fin R_f.

   (e) The pin fins then form a 5×5 array mounted on a base plate with a side length of s = 3 cm, as shown in the figure on the right-hand side. Calculate the overall efficiency of the fin array η_o and the thermal resistance of the fin array R_o. Calculate how much heat is dissipated through the unfined surface and how much is through the fins.



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3. (10 pt) Complete the portfolio assignment.
   Please select one homework assignment from this course that you found most valuable or educational. Write a 1-2 paragraph reflection discussing what you learned from that assignment

and how it will be useful to you. Next, create a new project page in your MAE portfolio repository on GitHub. Upload a PDF of your chosen homework assignment along with your reflection to this portfolio project page.

Then, complete the portfolio assignment submission. Please navigate to Canvas → Assignments → Portfolio Assignment → Submit Portfolio Project Link. Make sure to publish your page and provide the link to the published project page. The URL should look like: https://cornell-mae-ug.github.io/sp26-portfolio-yourgithubusername/projects/projectname/. It's not (a) a link to the GitHub repository (b) a link to your overall portfolio homepage.

If you already have the MAE portfolio, all you have to do is adding another page to your portfolio for this assignment and submit portfolio project link. If you do not have a portfolio, you can get started by accepting the invitation to make your personal repository at the following link: https://classroom.github.com/a/RQnJJnFr

For this problem, please attach screenshots of both the published MAE portfolio page and the portfolio assignment submission interface on Canvas to get the points. Please note: Completing the "Portfolio Assignment" on Canvas is required. The grade of uploading portfolio will be treated as part of the Problem Set grade, instead of the 5 points shown on Canvas. If you have any questions, please feel free to contact the TAs.