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Date of submission: 3/3/26

Problem Set Number: #4

Due Date: 3/3/26

Acknowledgement:

No help

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Known:

$L_g = 3mm = .003m$, $k_g = 1.5\,W/(mK)$

$L_a = .1mm = 1\times10^{-4}m$, $k_a = 1\,W/(mK)$

$R_c = 10^{-4}\,m^2K/W$

$G = 800\,W/m^2$

$T_\infty = T_{sur} = 300K$

$h = 10\,W/(m^2K)$

$h_{rad} = 5\,W/(m^2K)$

$a = .533$ $b = .001K^{-1}$

Find:

A) Thermal resistance + assumptions + each resistance value

B) $T_{g,top}$, T_{Si}

C) η and electric power flux q_{elec}

Assumptions:

1. Steady state
2. 1-D conduction through layers, uniform properties
3. Silicon is "very thin" → represented by a node at T_{Si}
4. Backside insulated → no heat loss from backside, all heat leaves the top surface.
5. Radiation modeled via effective coefficient h_{rad}

A

$$R_g = \frac{L_g}{k_g} = \frac{.003}{1.5} = 2\times10^{-3}$$

$$R_a = \frac{L_a}{k_a} = \frac{1\times10^{-4}}{1} = 1\times10^{-4}$$

Series conduction from silicon to top surface

$$R_{cond} = R_c + R_a + R_g = .0001 + .0001 + .0020 = 2.2\times10^{-3}$$

Top surface heat loss:

$$h_{tot} = h + h_{rad} = 10 + 5 = 15\,W/(m^2K) \rightarrow R_{surf} = \frac{1}{h_{tot}} = \frac{1}{15} = 6.67\times10^{-2}$$

$$\boxed{R_g = 2\times10^{-3},\ R_a = 1\times10^{-4},\ R_c = 1\times10^{-4},\ R_{cond} = 2.2\times10^{-3},\ R_{surf} = 6.67\times10^{-2}}$$

B

Solar at glass top:

$$q_{g,abs} = .1G = .1(800) = 80\,W/m^2$$

Solar Silicon:

$$q_{Si,abs} = .85G = .85(800) = 680\,W/m^2$$

Electrical conversion:

$$\eta = a - bT_{Si} = .553 - .001T_{Si}$$

Thermal generated in Silicon:

$$q_{Si,th} = (1-\eta)q_{Si,abs}$$

Total heat that must leave from the top surface:

$$q = q_{g,abs} + q_{Si,th} = 80 + (1-\eta)680$$

Top surface energy balance:

$$q = h_{tot}(T_{g,top} - T_\infty) \rightarrow T_{g,top} = T_\infty + \frac{q}{h_{tot}}$$

Conduction from silicon to top surface:

$$q = h_{tot}(T_{g,top} - T_\infty) \rightarrow T_{g,top} = T_\infty + \frac{q}{h_{tot}}$$

Conduction from silicon to top surface:

$$T_{Si} = T_{g,top} + qR_{cond}$$

$$\boxed{T_{Si} = 342.48K \quad T_{g,top} = 341.12K}$$

C

$$\eta = .553 - .001(342.48) = .2105$$

$$q_{elec} = \eta q_{Si,abs} = .2105(680) = 143.15\,W/m^2$$

$$\boxed{\eta = .2105\ (\approx 21.1\%)} \quad \boxed{q_{elec} = 143.15\,W/m^2}$$

② Known:

- $D = 10mm = .01m$, $k = 80\,W/(mK)$
- Wall temp: $T_w = 200°C$
- Insulation thickness: $L_{ins} = 200mm = .2m$
- Ambient: $T_\infty = 20°C$, $h = 20\,W/(m^2K)$
- Tip insulated

Find:

A) $T_0(L_0)$ expression

B) T_0 for $L_0 = .2m$ + check $T_{max} = 100°C$

C) min. L_0 for $T_0 \le 100\,C°$

Assumptions:

1. Steady state, 1D conduction along rod in insulated region.
2. Exposed portion behaves as a straight fin with adiabatic tip
3. Constant k, uniform h.

(A) $A = \frac{\pi D^2}{4}$, $P = \pi D$

$m = \sqrt{\frac{hP}{kA}}$

$q = \frac{kA}{L_{ins}}(T_w - T_0) \rightarrow q = \sqrt{hPkA}\,(T_w - T_0)$

$C = L_{ins} m \tanh(mL_0)$

$T_0 = \frac{T_w + CT}{1+C} \rightarrow \boxed{T_0(L_0) = \frac{T_w + (L_{ins} m \tanh(mL_0))T_\infty}{1 + L_{ins} m \tanh(mL_0)}}$

(B) $L_0 = .2m$

$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{h(4/D)}{k}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(20)}{80(.01)}} = 10$

$C = L_{ins} m \tanh(mL_0) = .2(10)\tanh(10 \cdot .2) = 2\tanh(2) = 1.928$

$T_0 = \frac{200 + 1.928(20)}{1 + 1.928} = 81.45°C$

$\boxed{T_0 = 81.45°C \rightarrow T_0 < T_{max} = 100°C}$ meets limit

(C)

$T_0 = 100°C$

$T_0 = \frac{T_w + CT_\infty}{1+C} \rightarrow C = \frac{T_0 - T_w}{T_\infty - T_0} = \frac{100-200}{20-100} = 1.25$

$C = L_{ins} m \tanh(mL_0) = .2(10)\tanh(10L_0) = 2\tanh(10L_0)$

$2\tanh(10L_0) = 1.25 \rightarrow \tanh(10L_0) = .625$

$10L_0 = \operatorname{arctanh}(.625) = .73317 \rightarrow L_0 = .07332$

$\boxed{L_{0,min} = .0733m \ (\approx 73.3mm)}$

3

Known:
- Copper fin: $k = 400\ W/(mK)$
- Square width: $s = 10\ mm = .01\ m$
- $h = 100\ W/(m^2K)$
- $\eta_f = .6$
- Convection tip

Find:
A) Fin length L

B) Fin thermal resistance R_f and fin effectiveness ε_f

Assumptions:
1. Straight fin, uniform cross-section, constant k, uniform h.
2. Use corrected length method for convection tip: $L_c = L + \frac{A_c}{P}$

Geometry and fin parameter

$$A_c = s^2 = (.01)^2 = 1 \times 10^{-4}\ m^2$$

$$P = 4s = 4(.01) = .04m$$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{100(.04)}{400(10^{-4})}} = 10\ m^{-1}$$

Corrected Length:

$$L_c = L + \frac{A_c}{P} = L + \frac{10^{-4}}{.04} = L + .0025$$

A

$$\eta_f = \frac{\tanh(mL_c)}{mL_c} = .6$$

$x = mL_c$

$\tanh(x)/x = .6 \rightarrow x = 1.51222$

$$L_c = \frac{x}{m} = \frac{1.51222}{10} = .15122\ m$$

$$L = L_c - \frac{A_c}{P} = .15122 - .0025 = .014872\ m$$

$$\boxed{L \approx 149\ mm}$$

B

$$A_f = PL + A_c = (.04)(.14872) + 10^{-4} = .0060489\ m^2$$

$$q_f = \eta_f h A_f (T_b - T_\infty) \rightarrow R_f = \frac{T_b - T_\infty}{q_f} = \frac{1}{\eta_f h A_f}$$

$$R_f = \frac{1}{(.6)(100)(.0060489)} = 2.755\ K/W$$

$$\varepsilon_f = \frac{q_f}{hA_c(T_b - T_\infty)} = \eta_f \frac{A_f}{A_c} = .6\frac{.0060489}{10^{-4}} = 36.29$$

$$\boxed{R_f = 2.76\ K/W \quad \varepsilon_f = 36.3}$$