Title: From Beat to Blood Flow

Topic of Interest: The Human Heart

Abstract: The system being studied in this project is the human heart, with a specific emphasis on how drugs and diseases affect the operation of the heart. This topic was chosen because it should be interesting to look at and learn about a system that is, on the surface, very different than the usual systems studied in this class. Additionally, it should be interesting to show how a biological system, such as the heart, can be modeled in similar ways to the mechanical and electrical systems studied in this class. The specific focus on drugs and disease aims to show how the heart system gets disrupted by natural effects, such as changes in contractility, heart rate, or vascular resistance, and is then stabilized or compensated for by medical technology and pharmaceutical interventions. The project will examine how these changes influence key outputs, such as blood pressure and cardiac output, and how the overall system dynamics (stability, transient response, and steady-state behavior) are affected. Additionally, the project will investigate whether drugs and diseases should be characterized as disturbances or control inputs in the model, in order to understand better what is actually happening within the feedback structure of the cardiovascular system and how best to represent real-world clinical interventions in a system dynamics framework.

Students/Roles:

Student	**Task/Role**	**Portfolio**
Name	1-3 sentence summary of student's work and key take-aways	n/a submitted through canvas
Ishan Roy	Worked on the system model and state space simulation section. Came up with an ODE and state space model for the system using researched parameters. Wrote a MATLAB script to simulate the ODE and the state space model. Was able to use the concepts of this class to model a novel system.	n/a submitted through canvas
Benjamin Brown	Researched conditions and diseases which affected the cardiovascular system. Investigated effects on blood pressure and health risks and verified that model reflected those effects. Overall, used skills and concepts learned in system dynamics to investigate and gain insight on real-world effects of common conditions on the cardiovascular system.	n/a submitted through canvas
Ravi Myers	Researched the effects of pressure and oxygen saturation on heart rate and blood pressure, tying them back to the ODE and state space model developed above. Overall worked with the ODE and Statespace models.	Submitted through canvas
Alex Shuey	Researched drugs that affect the cardiovascular system. Researched effects of heart rate reducing and increasing drugs and added a control input to the ODE that reflected those effects. Used skills and concepts learned in 3260 to put biological effects into a modelling equation.	n/a submitted through canvas

List of MAE 3260 concepts or skills used in this group work:

- ODEs and State Space models

System Model and State Space Simulation

To model the human heart using the tools and analysis of MAE 3260, a simple first-order ordinary differential equation was developed. Several system characteristics and parameters were used to effectively model the relationship between effective pumping rate (heart rate) and the mean arterial pressure (blood pressure). Heart rate was considered the input to the system, and pressure was considered the output. The reason for this was to study how different drugs and diseases affect the heart's ability to pump blood. Using this model, step changes in heart rate could be modeled, and the effects of the drugs and diseases could be analyzed by studying the differences in the response of the heart to a step change in desired heart rate. Four system characteristics were used in this model, which included the arterial compliance (C_a), the peripheral resistance (R_P), the mean venous pressure (P_v), and the stroke volume (SV). These four characteristics are all physical quantities whose values come from an individual's circulatory system. They are affected by drugs and diseases, and are therefore interesting to study in the context of this project.

The ordinary differential equation was developed by treating the systemic arteries as a single compliant chamber that stores blood volume and produces pressure. The rate of change of arterial pressure is linked to the rate of change of volume in this chamber, through the compliance C_a. Inflow to the arteries is taken to be the cardiac output, $Q_{in} = SV \cdot u(t)$, where SV is stroke volume, and u(t) is the heart rate. Outflow is affected by the peripheral resistance, which affects how blood actually flows through the arteries, so Qout = (Pa – Pv) / Rp. Applying conservation of volume, $Ca \cdot dPa/dt = Qin - Qout$, and substituting these expressions for inflow and outflow leads directly to the first-order ODE that governs how mean arterial pressure responds to changes in heart rate and vascular properties. The reason volume must be conserved is that for a control volume such as the arterial system, conservation of mass requires that the rate of change of mass inside the control volume equals the mass flow entering minus the mass flow leaving. Blood is modeled as an incompressible fluid, meaning its density is assumed constant. Under this assumption, mass conservation directly reduces to volume conservation, since mass equals density times volume and density does not change.

$$\frac{dPa}{dt} = -\frac{1}{R_p * C_a} Pa(t) + \frac{P_V}{R_p * C_a} + \frac{SV}{C_a} u(t)$$

First-order ODE of the system

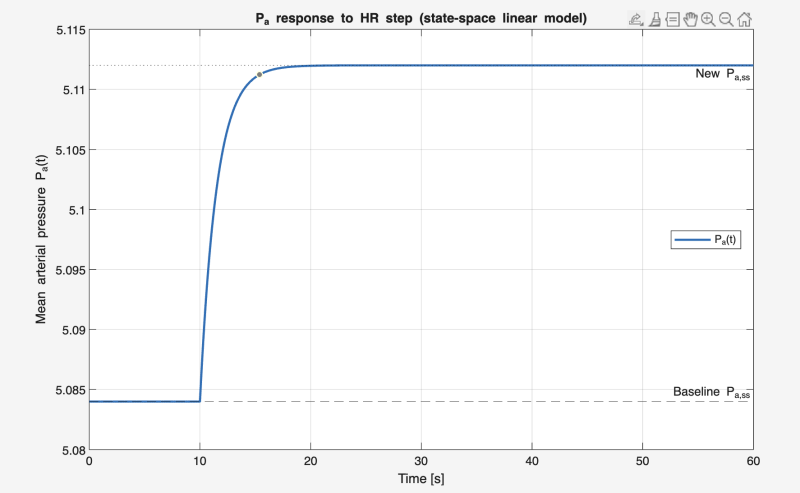
Using this ODE, a state-space model was also developed in order to simulate the system in MATLAB. It was obtained by taking the ODE and then expressing it in terms of deviations from a steady state point. The original equation relates the rate of change of arterial pressure to the current arterial pressure, venous pressure, and heart rate inputs, but it includes a constant

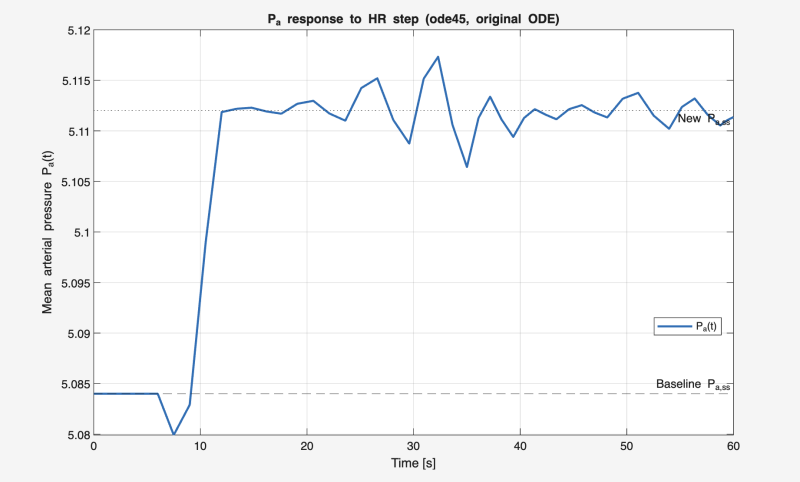
offset that depends on the chosen baseline heart rate and the venous pressure. To remove this constant term and obtain a linear time-invariant model, a steady state is first identified for a nominal heart rate, giving a corresponding steady arterial pressure. The pressure state is then redefined as a deviation from this steady value, and the input is similarly expressed as a deviation from the nominal heart rate. Substituting these deviation variables into the ODE and using the steady-state condition to cancel the constant terms yields a simple first-order state equation in which the rate of change of the deviation state depends linearly on the deviation state and the deviation input. The output is then the deviation in pressure from the steady state value.

$$x(t) = P_a(t) - P_{a,ss} \qquad w(t) = u(t) - u_0$$

$$\dot{x}(t) = \frac{-1}{R_p C_a} x(t) + \frac{SV}{C_a} w(t) \; ; \quad y(t) = x(t) + P_{a,ss}$$

State Space Model of the System





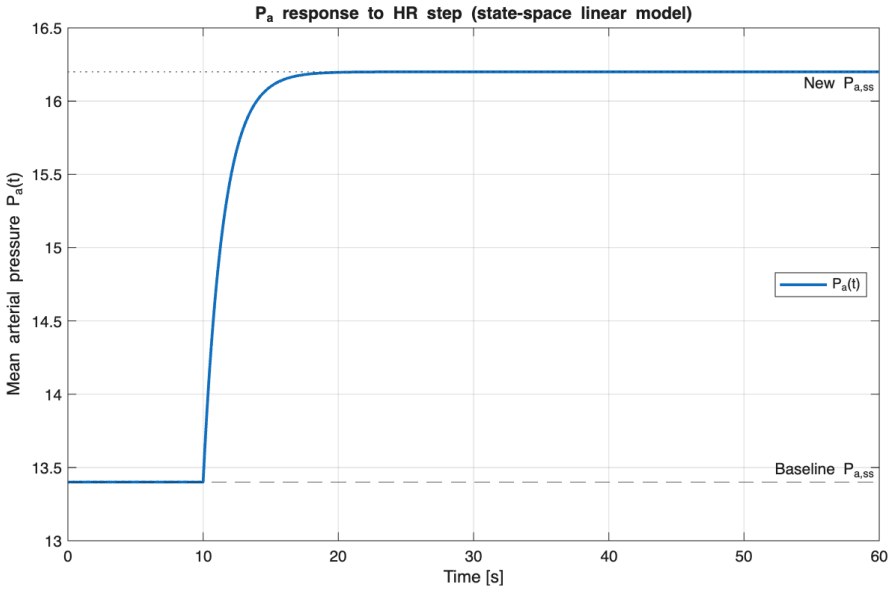
MATLAB was then used to perform several simulations using this state space model and the original equation. The simplified state-space model yields a clean, linear response, whereas the ODE 45 simulation more accurately represents the original model. The results of a simulation using nominal values are shown here. The linear state-space model exhibits a smooth response, showing how the heart responds to a necessary increase in heart rate (as indicated in the graph by a step input to arterial pressure). The original ODE simulation shows the fluctuations in the response as the system tries to stabilize at the reference values. This instability shows up due to the numerical methods being used to solve the equation.

How Cardiovascular Diseases Affect the Model

Hypertension, colloquially known as high blood pressure, affects nearly half of the adult population in the United States [4]. It arises from various diseases, some of which do not specifically affect the heart or its arteries, but do still have secondary effects on them. Hypertension, and other diseases that affect the cardiovascular system, can be explored, and their effects understood better, if viewed using a model which takes into account various parameters, especially those mentioned above (peripheral resistance, arterial compliance, and venous pressure). This section will categorize various diseases based on their effect on those parameters, and then explain how viewing them using a system dynamics perspective furthers our understanding of those diseases and the threat they pose to the human body.

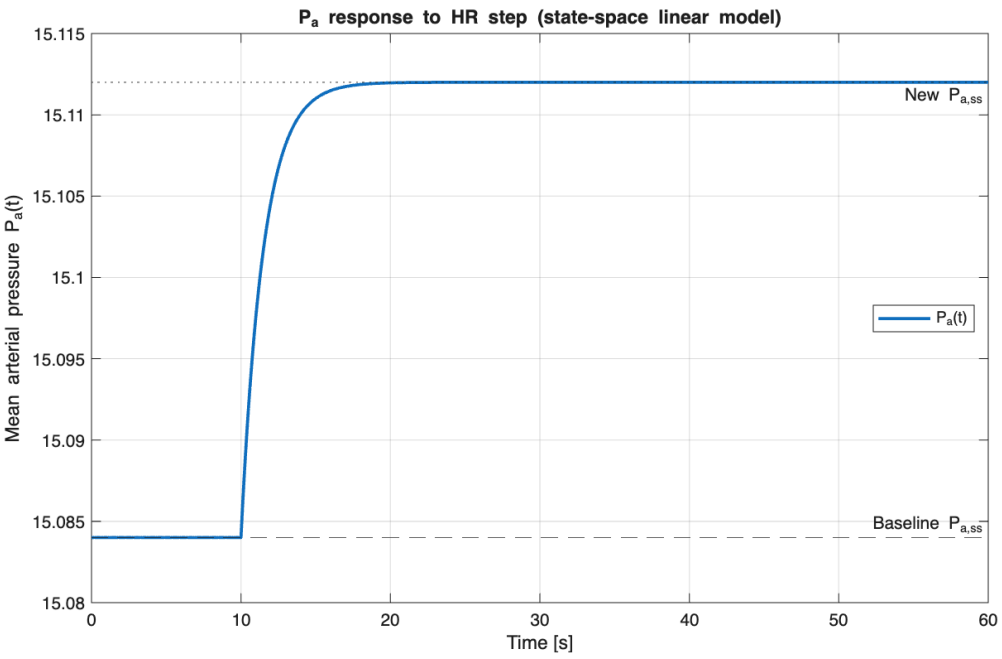
Peripheral resistance is defined as a coefficient within the governing equation which represents resistance in the body's blood vessels which slow, block, or otherwise constrict the flow of blood throughout the body. Hyperlipidemia, or high cholesterol, is an example of a disease which would specifically impact this parameter. Hyperlipidemia's main effect and risk factor comes in the form of its potential to build up plaque in the arteries (called atherosclerosis) from excess lipids in the body, which build up over time [5]. This specifically does not have a substantial effect on blood pressure, but the plaque, once large enough, can break off and become stuck in the blood vessels, blocking blood flow through that specific blood vessel. If that blood vessel happens to be a primary vein carrying blood towards the brain and it is blocked, this results in a stroke.

Investigating the risk of stroke specifically, the model correctly predicts that if the arterial compliance C_a is substantially lower than normal and the peripheral resistance R_p is drastically higher than normal, the steady state arterial pressure behind the clot increases drastically. This is expected, as the blood is blocked but the heart continues pumping blood into that vein, drastically increasing blood flow behind the clot while decreasing it ahead of the clot. Assuming that peripheral resistance increases by a similar factor as arterial compliance decreases by, the time constant of the system does not change much but the steady state pressure behind the blood clot increases drastically. Shown above is a plot which represents that, as you can see the steady state behavior is far



higher than normal, but if there is a blood clot this requires that the pressure on the other side of the clot compensates by being extremely low, indicating low oxygen ahead, specifically towards the brain.

Regarding hypertension, two of the three leading causes are obesity and consuming a diet with excess sodium [6]. The excess sodium causes the body to release more water into the bloodstream, increasing the density and blood pressure. Obesity's relationship with hypertension has more to do with the nervous system, renin-angiotensin system, and sodium retention [7]. The first two matters fall outside the scope of the model, but the third is related to increased blood pressure in the same manner that sodium-high diet is. The condition has symptoms of arterial pressure throughout the body at levels substantially higher than normal. Diseases which affect the body in such a way, causing greater amounts of fluid in the blood than what would be healthy otherwise, are represented well by the model through the adjustment of the venous pressure parameter. Increasing it affects the model by increasing both the resting arterial pressure and steady state arterial pressure when heart rate increases, which represents hypertension well, elevating the body's resting blood pressure. See graph on left.

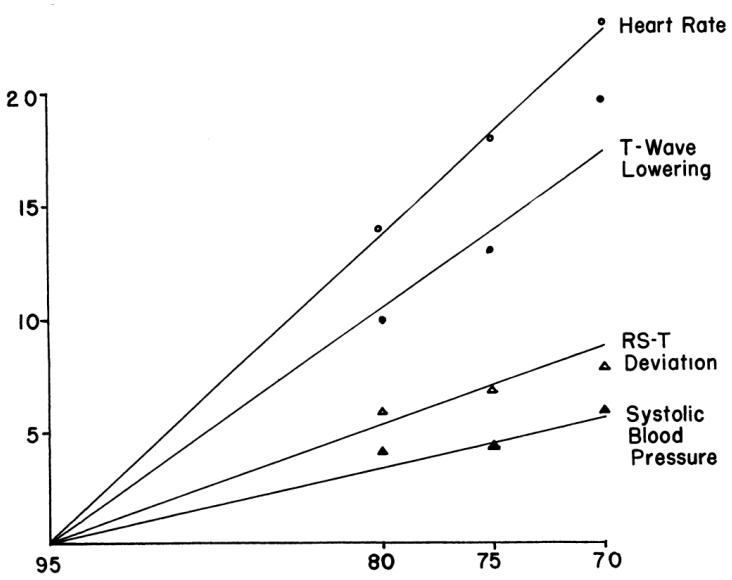


Other conditions and diseases could also be well simulated by the model, such as shock after physical trauma, aging, or even high sugar intake, which has a similar effect to increased cholesterol levels. Both examples above correctly model the heart's ability to adjust to changing parameters, but these changes in steady state behavior have long lasting effects on the heart, resulting in the diseases that arise from hypertension observed globally. The model's parameters can be used to effectively simulate the effects of common diseases in human populations, especially hypertension, and also helps to develop an intuition for how those diseases affect the bloodstream and heart strength.

The Effects of Oxygen Saturation and Pressure Change on Heart Rate

A person's heart rate is a measure of how fast their blood is flowing, which increases or decreases depending on the oxygen requirements of the body. Oxygen saturation therefore is a vital statistic to heart rate, as when oxygen saturation of the blood decreases, the heart rate must increase to equalize the rate of oxygen transference to cells. Exploring the relationship between oxygen saturation and heart rate gives vital information on the functions of the heart, which can be used for synthetic aids for heart problems.

Oxygen saturation, often measured as a percentage, provides a new variable to any form of state-space model, one that scales the entire equation. To determine what form of modification the oxygen saturation term fulfills, detailed study must be conducted, and the results of such a study are displayed on the right.[1] As can be seen, the heart rate of the observed patients increased linearly as the oxygen saturation (The X variable for this study) was decreased. Borrowing the ODE from earlier,



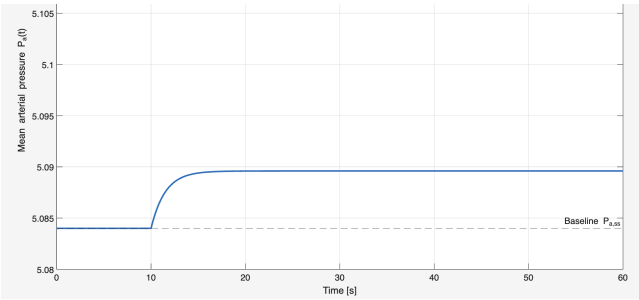
$$\frac{dPa}{dt} = -\frac{1}{R_p * C_a}Pa(t) + \frac{P_V}{R_p * C_a} + \frac{SV}{C_a}u(t)$$

We can modify it in such a way that the u(t) is divided by the oxygen saturation, which will then equivalate to a linear relationship of heart rate to oxygen saturation which will look like

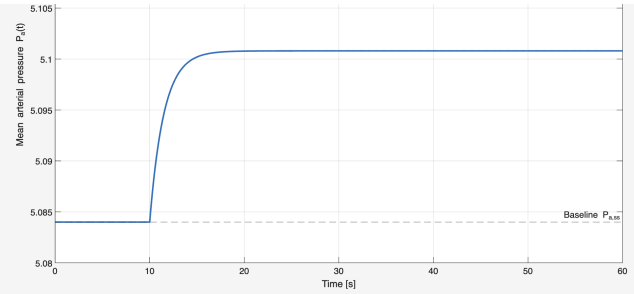
$$\frac{dPa}{dt} = -\frac{1}{R_p * C_a}Pa(t) + \frac{P_V}{R_p * C_a} + \frac{SV}{C_a O_S}u(t)$$

with O_s being the oxygen saturation of the blood.

An example of this change in the resultant blood pressure can be seen on the right (from 80% to 90%), as the lower effect of the heart rate upon blood pressure causes it to dip. Notably, this assumes that heart rate stays constant, and so the body would instead raise heart rate so blood pressure gains to its previous value. As such, regardless of the oxygen saturation, blood pressure



shouldn't be affected by any disturbances of oxygen saturation, however heart rate would be.



Next we can look at the differences in heart rate from pressure changes, which occur in high altitude hiking. Differences in pressure from high altitude changes the pressure exerted on the lungs, and on the body overall as well as the oxygen saturation of the air itself, changing the heart rate through oxygen saturation of the blood. As a result altered pressure can cause changes in blood pressure and heart rate, which is important to diagnose those sensitive to pressure changes who should not be going on hikes up to higher elevations.

We can see the effect of the change in pressure in the study published by The American Journal of Emergency Medicine[2], which shows a market difference in both the oxygen saturation, as well as the heart rate, and blood pressure. Following the rise of 3.6 km, the subjects heart rate increased by an average of 37, their oxygen saturation dipped by 18%, yet also became far more variable, and their blood pressure increased by an average of 11. Using the previous study as a bench mark for oxygen saturation's relationship to heart rate, we can note that the decrease in O_2 saturation would correlate with a reduction of 15 bpm of heart rate, yet in this study we see an excess of 22 bpm, the remainder of which likely come from the change in blood pressure, as lowered oxygen saturation leads to the autonomic nervous and the endocrine systems activating, and higher blood pressure, as a method of making more atoms of oxygen transfer for every so much volume of blood that is moved.

	760 mm Hg	**495 mm Hg**
SpO_2 (%)	98 ± 1	80 ± 5**
Heart rate (beats/min)	70 ± 10	107 ± 8*
Mean blood pressure (mm Hg)	90 ± 12	101 ± 16

To accommodate this into our ODE, we can speculate on the relation that pressure has upon the heart rate and blood pressure of subjects, however we cannot be certain, as for any kind of relationship for the two points that we have will not satisfy anything except a linear relationship.

The Effects of Drugs on Heart Rate

Heart-rate–modifying drugs can be interpreted as external control inputs that either decrease or increase the effectiveness of the heart's pumping action. Heart-rate–reducing drugs, such as beta-blockers and non-dihydropyridine calcium-channel blockers, act like negative control inputs that blunt the system's response to the normal heart-rate command signal. Beta-blockers reduce the gain on the heart's "actuator" by blocking sympathetic stimulation, so the same control effort produces a smaller change in heart rate and therefore a smaller pumping effect. Calcium-channel blockers similarly dampen the dynamic response by slowing conduction through the AV node, effectively increasing the system's resistance to changes in heart rate. Together, these drugs reduce how strongly the control input can influence the state, pulling the arterial-pressure trajectory toward a lower steady-state value.

Heart-rate–increasing drugs, such as β-agonists and anticholinergics, behave like positive control inputs that inject additional effort into the system. β-agonists act as an added excitatory signal to the heart, increasing its responsiveness and boosting the effective actuator output for a given control command. Anticholinergics work by removing parasympathetic braking, which functions like reducing a negative feedback loop in the control architecture; once that inhibitory pathway is suppressed, the heart responds more strongly to any input that would raise its rate. These drugs enhance the system's control authority, pushing the arterial-pressure state toward a higher steady-state level and producing a faster dynamic response to inputs.

1st Order ODE (with heart rate reducing drugs)

$$\frac{dPa}{dt} = -\frac{1}{R_p * C_a}P_a(t) + \frac{P_V}{R_p * C_a} + \frac{SV \cdot HR_0}{C_a}u(t) - kdrug \cdot uHR(t)$$

1. Beta-blockers (e.g., Metoprolol, Atenolol)
These block β_1-adrenergic receptors in the heart, reducing sympathetic stimulation. That slows SA-node firing, reduces heart rate, and decreases contractility. Effectively reduce the pumping term tied to HR.

2. Calcium-channel blockers (non-dihydropyridines, e.g., Verapamil, Diltiazem)
These act on the AV node to slow conduction and lower heart rate. They blunt the pumping response and decrease myocardial oxygen demand.

1st Order ODE (with heart rate increasing drugs)

$$\frac{dPa}{dt} = -\frac{1}{R_p * C_a}P_a(t) + \frac{P_V}{R_p * C_a} + \frac{SV \cdot HR_0}{C_a}u(t) + kdrug \cdot uHR(t)$$

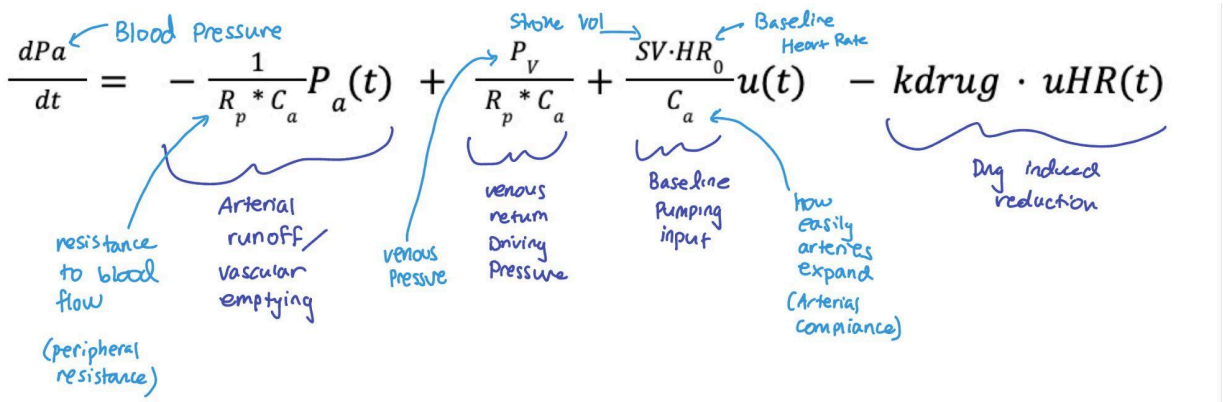
1. β-agonists (e.g., Isoproterenol, Dobutamine)

These stimulate β_1-adrenergic receptors in the heart, increasing SA-node firing and contractility. Clinically used in shock, severe bradycardia, or heart block.

2. Anticholinergics (e.g., Atropine)

These block parasympathetic (vagal) input at muscarinic receptors, removing the "brake" on the SA node. Commonly used to treat symptomatic bradycardia.

Labelled ODE:



Citations :

[1]: PENNEYS, RAYMOND, and CAROLINE BEDELL THOMAS. “The Relationship between the Arterial Oxygen Saturation and the Cardiovascular Response to Induced Anoxemia in Normal Young Adults.” *Circulation*, vol. 1, no. 3, Mar. 1950, pp. 415–425, https://doi.org/10.1161/01.cir.1.3.415.

[2]: Shigeru Saito, Kyoko Tanobe, Makiko Yamada, Fumio Nishihara, Relationship between arterial oxygen saturation and heart rate variability at high altitudes, The American Journal of Emergency Medicine, Volume 23, Issue 1, 2005, Pages 8-12, ISSN 0735-6757, https://doi.org/10.1016/j.ajem.2004.09.023.

[3]: Kamiński, Marek, et al. “Evaluation of the Impact of Atmospheric Pressure in Different Seasons on Blood Pressure in Patients with Arterial Hypertension.” International Journal of Occupational Medicine and Environmental Health, vol. 29, no. 5, 22 July 2016, pp. 783–792, https://doi.org/10.13075/ijomeh.1896.00546. Accessed 7 Aug. 2020.

[4] U.S. Centers for Disease Control and Prevention, “High Blood Pressure Facts,” *U.S. Centers for Disease Control and Prevention*, 2025. [Online]. Available: https://www.cdc.gov/high-blood-pressure/data-research/facts-stats/index.html. [Accessed: Dec. 9, 2025].

[5] Cleveland Clinic, “High Cholesterol Diseases,” *Cleveland Clinic*, 2022. [Online]. Available: https://my.clevelandclinic.org/health/articles/11918-cholesterol-high-cholesterol-diseases. [Accessed: Dec. 9, 2025].

[6] University of Utah Health, “What causes high blood pressure?” *University of Utah Health*. [Online]. Available: https://healthcare.utah.edu/cardiovascular/conditions/hypertension/causes. [Accessed: Dec. 9, 2025].

[7] National Library of Medicine; National Center for Biotechnology Information, “Obesity and hypertension,” 2016 Sep 6. [Online]. Available: https://pmc.ncbi.nlm.nih.gov/articles/PMC5038894/. [Accessed: Dec. 9, 2025].

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%   dPa/dt = -1/(Rp*Ca)*Pa + Pv/(Rp*Ca) + (SV/Ca)*u(t)
% where u(t) = heart rate (HR)


% PARAMETERS
Rp  = 1.0;        % peripheral resistance
Ca  = 1.5;        % arterial compliance
Pv  = 5.0;        % venous pressure
SV  = 70e-3;      % stroke volume

HR0 = 1.2;        % baseline heart rate u0 [beats/s]
HR1 = 1.6;        % new heart rate after step [beats/s]


% Derived system quantities
tau   = Rp*Ca;                      % time constant of arterial compartment
Pa_ss0 = Pv + Rp*SV*HR0;            % steady-state Pa at baseline HR
Pa_ss1 = Pv + Rp*SV*HR1;            % steady-state Pa after HR step

fprintf('Time constant tau = %.2f s\n', tau);
fprintf('Baseline MAP P_a,ss = %.2f\n', Pa_ss0);
fprintf('New MAP P_a,ss after HR step = %.2f\n\n', Pa_ss1);


% STATE-SPACE SIMULATION
% We define x = Pa - Pa_ss0, w = u - HR0
% Then: dx/dt = A x + B w,  y = x
% with A = -1/(Rp*Ca), B = SV/Ca

A = -1/(Rp*Ca);
B =  SV/Ca;
C = 1;
D = 0;

sys = ss(A,B,C,D);

% time vector
t  = 0:0.1:60;      % simulate 60 seconds

% input: HR is HR0 for t<10, HR1 for t>=10
u  = HR0*ones(size(t));
u(t>=10) = HR1;

% deviation input w = u - HR0
w = u - HR0;

% initial condition for deviation state x(0) = Pa(0) - Pa_ss0
x0 = 0;                         % start at steady state (Pa(0) = Pa_ss0)

% simulate deviation in pressure
[y_dev, t_out, x_dev] = lsim(sys, w, t, x0);
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```
% actual arterial pressure
Pa = y_dev + Pa_ss0;

% Plot results
figure;
plot(t_out, Pa, 'LineWidth', 1.8);
hold on;
yline(Pa_ss0,'--','Baseline P_{a,ss}');
yline(Pa_ss1,':','New P_{a,ss}','LabelVerticalAlignment','bottom');
xlabel('Time [s]');
ylabel('Mean arterial pressure P_a(t)');
title('P_a response to HR step (state-space linear model)');
legend('P_a(t)','Location','best');
grid on;


% DIRECT ODE SIMULATION WITH ode45
%   dPa/dt = -1/(Rp*Ca)*Pa + Pv/(Rp*Ca) + (SV/Ca)*u(t)

% define heart rate as a function of time
u_fun = @(t) (t < 10).*HR0 + (t >= 10).*HR1;

% ODE right-hand side
odefun = @(t,Pa) -1/(Rp*Ca)*Pa + Pv/(Rp*Ca) + (SV/Ca)*u_fun(t);

% initial condition: start at Pa_ss0
Pa0 = Pa_ss0;
tspan = [0 60];

[t_ode, Pa_ode] = ode45(odefun, tspan, Pa0);

% Plot comparison
figure;
plot(t_ode, Pa_ode, 'LineWidth', 1.8);
hold on;
yline(Pa_ss0,'--','Baseline P_{a,ss}');
yline(Pa_ss1,':','New P_{a,ss}','LabelVerticalAlignment','bottom');
xlabel('Time [s]');
ylabel('Mean arterial pressure P_a(t)');
title('P_a response to HR step (ode45, original ODE)');
legend('P_a(t)','Location','best');
grid on;
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```
Time constant tau = 1.50 s
Baseline MAP P_a,ss = 5.08
New MAP P_a,ss after HR step = 5.11
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