Kasper Atkinson (kna37)

#1

S=3cm 5x5 array

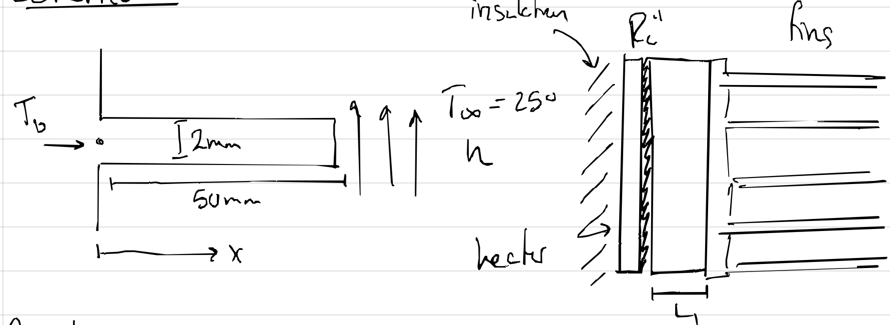
Known: Cylindrical pin $r = 1mm$ $l = 5cm$ exposed to $T_\infty = 25°C$. Heat stab $L_1 = 2cm$ with flux $= 1000 \frac{W}{m^2}$. Thermal adhesive.

Find: a) $T_{\frac{L}{2}}$ & T_L if $T_b = 125°C$ b) ε_f & η_f 3.4 c) η_s 3.5. Discuss approx. d) Thermal resistance e) For array find η_o, R_o, surface & fin heat loss. f) Draw R network & find T_h. Compare fins & not fins. ΔT?

Assumptions: No radiation. Isolated tips. Heat sink $R = 0$

Properties: $h = 10 \frac{W}{m^2K}$ $k = 200 \frac{W}{mK}$, $R_c = 1E\text{-}4 \frac{W}{m^2}$, $k_1 = 10 \frac{W}{mK}$

Schematic:



Analysis:

a)

$$\theta = \theta_b \frac{\cosh\left(\sqrt{\frac{hP}{kA_c}}(L - x)\right)}{\cosh\left(\sqrt{\frac{hP}{kA_c}} L\right)}$$

$$\frac{P = 2\pi r}{A_c = \pi r^2} = \frac{2}{r}$$

$$T_x = (T_b - T_\infty) \frac{\cosh\left(\sqrt{\frac{hP}{kA_c}}(L - x)\right)}{\cosh\left(L\sqrt{\frac{hP}{kA_c}}\right)} + T_\infty$$

$$T_{\frac{1}{2}L} = (125 - 25) \frac{\cosh\left(\sqrt{\frac{10 \cdot 2}{200 \cdot 0.001}}\left(\frac{1}{2}(0.05)\right)\right)}{\cosh\left(0.05\sqrt{\frac{10 \cdot 2}{200 \cdot 0.001}}\right)} + 25 = \boxed{116.5°C}$$

$$T_L = (125 - 25) \frac{\cosh(0)}{\cosh\left(0.05\sqrt{\frac{10 \cdot 2}{200 \cdot 0.001}}\right)} + 25 = \boxed{113.7°C}$$

b) $\varepsilon_f = \frac{q_f}{hA_c\theta_b} = \frac{kA_c\cancel{\theta_b} m \tanh(mL)}{hA_c\cancel{\theta_b}} = \frac{k}{h}\sqrt{\frac{2h}{kr}} \tanh\left(\sqrt{\frac{2h}{kr}} L\right)$

$= \sqrt{\frac{2(200)}{10(0.001)}} \tanh\left(0.05\sqrt{\frac{2(10)}{200 \cdot 0.001}}\right) = \boxed{92.4}$

$\eta_f = \frac{q_f}{q_{max}} = \frac{kA_c\cancel{\theta_b} m \tanh(mL)}{hA_f\cancel{\theta_b}}$

$= \frac{200 \cdot \pi(0.001)^2}{10 \cdot 2\pi(0.001)(0.05)} \sqrt{\frac{2(10)}{200(0.001)}} \tanh\left(0.05\sqrt{\frac{2(10)}{200 \cdot 0.001}}\right) = \boxed{0.92423}$

c)

$L_c = \frac{V}{A} = \frac{LA}{A} = 0.05 m$

$\eta_f = \frac{\tanh\left(\sqrt{\frac{2h}{kr}} L_c\right)}{\sqrt{\frac{2h}{kr}} L_c} = \frac{\tanh\left(0.05\sqrt{\frac{2(10)}{200(0.001)}}\right)}{0.05\sqrt{\frac{2(10)}{200(0.001)}}} = \boxed{0.92423}$

Table 3.5 provides a very good approximation for efficiency when comparing to 3.4. They are pretty much identical.

d) $R_f = \frac{1}{hA_f\eta_f} = \frac{1}{10(2\pi(0.001)(0.05))(0.924)} = \boxed{344 \frac{K}{W}}$

e)

$\eta_o = 1 - N\frac{A_f}{A_{tot}}(1-\eta_f) = 1 - \frac{25(2\pi(0.001)(0.05)(1-0.924))}{(0.03)^2 - (25\pi(0.001)^2) + (25)(2\pi(0.001)(0.05))}$

$= \boxed{0.931}$

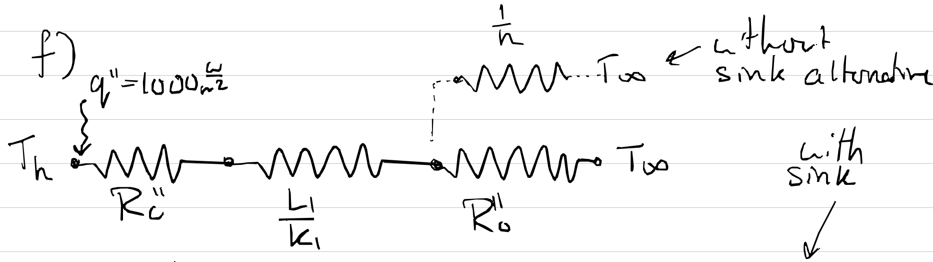
$R_o = \frac{1}{\eta_o h A_{tot}} = \frac{1}{(0.931)(10)[(0.03)^2 - (25\pi(0.001)^2 + (25(2\pi(0.001)(0.05))]}$

$= \boxed{12.4 \frac{K}{W}}$ $\quad q_{tot} = \frac{\theta_b}{R_o} = 8.07 W \quad q_f = 8.07 - 0.821 = \boxed{7.24 W}$

$q_b = h\theta_b A_b = (10)(100)[(0.03)^2 - (25\pi(0.001)^2)] = \boxed{0.821 W}$

base ↑

f)



$$R_{eff}'' = R_c'' + \frac{L_1}{k_1} + R_o S^2 = 1E{-}4 + \frac{0.02}{10} + 12.4(0.03)^2 = 13.3E{-}3 \ \frac{m^2 K}{W}$$

$$q'' = \frac{\Delta T}{R_{eff}''}$$

sink

$$q'' R_{eff}'' + T_\infty = T_h = 25 + 1000(13.3E{-}3) = \boxed{38.26\,^\circ C}$$

$$R_{eff}' = 1E{-}4 + \frac{0.02}{10} + \frac{1}{10} = 102E{-}3 \ \frac{m^2 K}{W}$$

$$T_h = 1000(102E{-}3) + 25 = \boxed{127.1\,^\circ C}$$

88.8 °C reduction in temp with the heat sink

#2

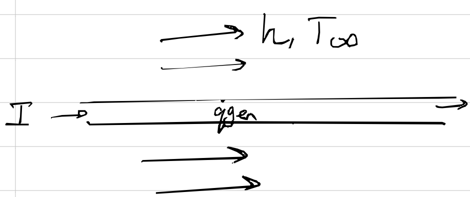
Known: $D = 1mm$, $T_\infty = 20°C$, $I = 100A$. $t = 0s \to \infty$

Find: a) Steady state temp. b) Transient temp, time to $5°$ s.s.

Assumptions: No radiation, infinitely long wire.

Properties: $\rho = 4000 \frac{kg}{m^3}$, $k = 400 \frac{W}{m \cdot K}$, $c = 500 \frac{J}{kg \cdot K}$, $R_e' = 0.01 \frac{\Omega}{m}$, $h = 500 \frac{W}{m^2 K}$

Schematic:



Analysis

a) $$-hP(T - T_\infty) + I^2 R_e' = \cancel{\rho A_c c \frac{\partial T}{\partial t}}^{\,0\ (s.s.)}$$

$$\frac{I^2 R_e'}{hP} + T_\infty = T = \frac{(100)^2 (0.01)}{500(0.001\pi)} + 20 = \boxed{83.7°C}$$

b) $$-hP(T - T_\infty) + I^2 R_e' = \rho A_c c \frac{\partial T}{\partial t}$$

$$-hP\theta + I^2 R_e' = \rho A_c c \frac{\partial \theta}{\partial t}$$

$$\rho A_c c \frac{\partial \theta}{\partial t} + hP\theta = I^2 R_e'$$

system dynamics first order ODE solution

$$\frac{\partial \theta}{\partial t} + \frac{hP}{\rho A_c c}\theta = \frac{I^2 R_e'}{\rho A_c c} \qquad \theta(0) = 0$$

wire is at bath temp?

$$\theta(t) = \frac{I^2 R_e'}{hP}\left(1 - e^{\frac{-hP}{\rho A_c c} t}\right)$$

$$T(t) = T_\infty + \frac{I^2 R_e}{hP}\left(1 - e^{\frac{-hP}{\rho A_c c} t}\right)$$

$$= 20 + \frac{100^2\, 0.01}{500(0.001\pi)}\left(1 - e^{\frac{-500(0.001\pi)}{4000(\pi(0.001)^2)(500)} t}\right)$$

$$83.7 - 5 = 20 + 63.7\left(1 - e^{-1t}\right) \rightarrow \boxed{t = 2.54s}$$

#3

Portfolio assignment. From Ed Discussion, no formatting needed.