Sibley School of Mechanical and Aerospace Engineering

MAE 3240 Heat Transfer

Spring 2026

Problem Set #3

Posted: Monday, Feb 9 **Due: Tuesday, Feb 24, 11:59 pm**

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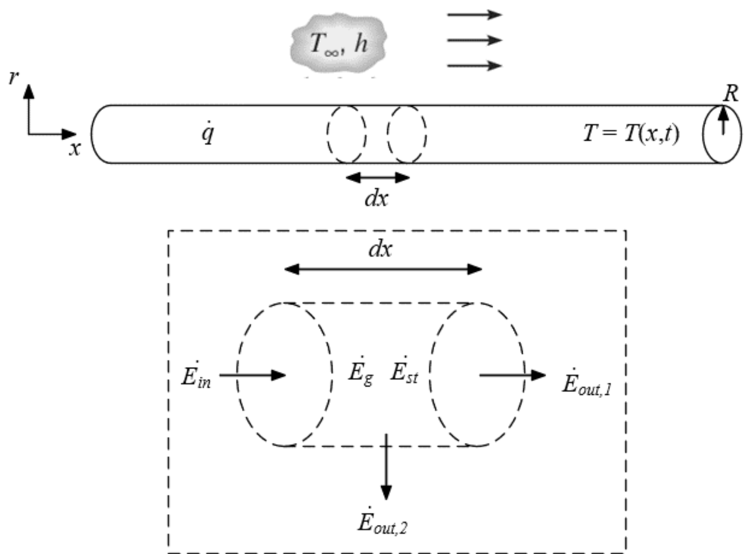
1. (30 pt) Consider a very long rod with a radius R and a uniform volumetric heat generation rate of $\dot{q}$. This rod is placed in an environment with ambient temperature T_∞ and heat transfer coefficient h. Thermal radiation can be neglected. We set a coordinate system with x-axis through the axis of the rod, as shown in the figure. Assume that the radius of the rod is very small and the temperature of the rod along the radial direction is uniform. As a result, the temperature of the rod is a function of the x coordinate and time t, i.e., $T = T(x,t)$. In addition, we can assume the thermal conductivity k, density ρ, and heat capacity c_p are constant. Please revisit the derivation of heat equation in class and answer the following two questions.

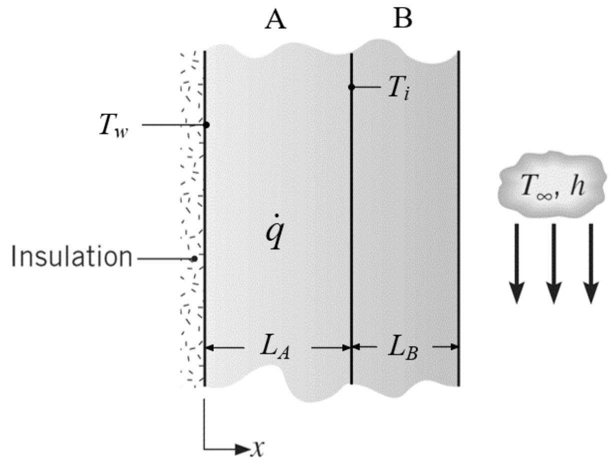


   (a) Consider a differential cylinder with a height of dx as the control volume. The figure above shows an energy balance diagram within the control volume. Please write down the energy balance equation for this control volume. Please then express each energy term in the energy balance equation as a function of T by choosing proper rate equations.

   (b) Derive the governing differential equation that describes the temperature profile of the rod and simplify it as much as possible.

   (c) This rod has a length of L. One end of the rod is located at $x = 0$ and maintained at a constant temperature of T_1. The other end of the rod is located at $x = L$ and exposed to the air flow with temperature of T_∞ and heat transfer coefficient h. At t = 0 s, the rod has a uniform initial temperature of T_i. Please write proper boundary conditions and initial conditions for the governing differential equation.

2. (40 pt) Consider one-dimensional heat conduction in a plane composite wall. Wall A has a thickness of L_A = 30 cm, which experiences uniform volumetric heat generation $\dot{q}$. The thermal conductivity of wall A is k_A. The left surface of wall A is thermally insulated. Wall B has a thickness of L_B = 10 cm and thermal conductivity k_B = 30 W/(m·K).



There is no heat generation within wall B. The left surface of wall B is in contact with wall A. There is no contact thermal resistance at their boundary. The right surface of wall B is exposed to a coolant at T_∞ = 20 °C and a convective heat transfer coefficient of 500 W/(m^2·K). We insert two temperature sensors to measure the surface temperatures of wall A. The sensor at the interface between the thermal insulation and wall A shows T_w = 115 °C. The other sensor at the interface between wall A and wall B shows T_i = 100 °C.

(a) Calculate the right surface temperature T_s of wall B.

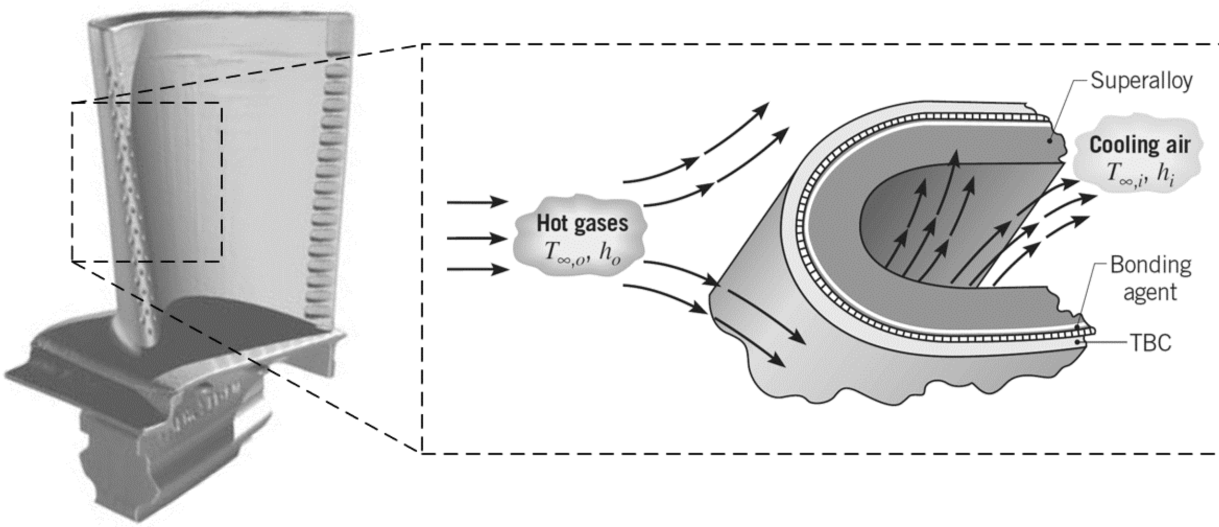
(b) Calculate the uniform volumetric heat generation $\dot{q}$ at steady state.

(c) Calculate the thermal conductivity k_A of wall A.

(d) Plot the temperature profile across wall A and wall B and show its important features.

3. (30 pt) Energy efficiency of a gas turbine can be improved by operating it under higher temperature. However, hot gases produced by the combustor pose a significant challenge to the reliable operation of turbine blades. An approach to protecting the turbine blades from material failure in the high-temperature environment is to apply a "thermal barrier coating (TBC)" to the exterior surface of the blade, as shown in the figure below. The blade is made from a high-temperature superalloy with thermal conductivity of k_b = 25 W/(m·K). The TBC

is made of a ceramic material with thermal conductivity of k_{TBC} = 1 W/(m·K). To attach the TBC to the exterior surface of the superalloy, a metallic bonding agent (thickness negligible) is needed, which creates an interfacial thermal resistance (per unit area) of R'' = 10^{-4} m^2·K/W. Consider an operating condition for which external hot gases at $T_{\infty,o}$ = 1616 K and cooling air inside the blade at $T_{\infty,i}$ = 400 K. Heat transfer coefficients of hot gases and cooling air are h_o = 1000 W/(m^2·K) and 500 W/(m^2·K), respectively. The turbine blade is at a steady-state condition, where thermal radiation can be neglected. The turbine blade can be approximated as a plane wall, so that we can assume one-dimensional heat conduction through the TBC and the superalloy. The thickness of superalloy blade is L_b = 5 mm. The maximum allowable temperature for the superalloy is T_{max} = 1200 K. Consider the following two conditions.



(a) Condition 1: the TBC is not applied. If we do not apply the TBC and directly expose the superalloy blade to hot gases, draw the thermal resistance network for this condition and label all thermal resistances. Calculate the exterior surface temperature of the superalloy blade. Can the superalloy blade be maintained below T_{max}?

(b) Condition 2: the TBC is applied. Now we apply L_{TBC} = 0.5 mm thick TBC to the superalloy blade. Draw the thermal resistance network for this condition and label all thermal resistances. Calculate the temperature of the superalloy blade again. Can the superalloy blade be maintained below T_{max} this time?