# Cover Page

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Problem set #3    Due date: 2/24/26

Acknowlegment: discussed briefly with Toby Huynh

(used the "Fundamentals of Heat & Mass Transfer" textbook for equations)

# #1 Known: Radius $R$, heat gen $\dot{q}$, $T_\infty$, $h$

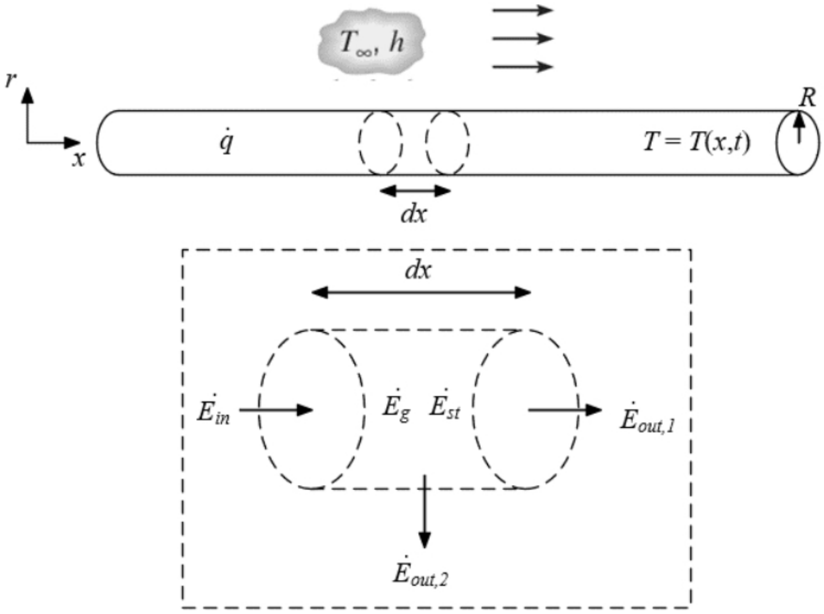
**Find:**

(a) Consider a differential cylinder with a height of $dx$ as the control volume. The figure above shows an energy balance diagram within the control volume. Please write down the energy balance equation for this control volume. Please then express each energy term in the energy balance equation as a function of $T$ by choosing proper rate equations.

(b) Derive the governing differential equation that describes the temperature profile of the rod and simplify it as much as possible.

(c) This rod has a length of $L$. One end of the rod is located at $x = 0$ and maintained at a constant temperature of $T_1$. The other end of the rod is located at $x = L$ and exposed to the air flow with temperature of $T_\infty$ and heat transfer coefficient $h$. At $t = 0$ s, the rod has a uniform initial temperature of $T_i$. Please write proper boundary conditions and initial conditions for the governing differential equation.

**Schematic:**




**Assumptions:**

$T = T(x,t)$

$k$, $\rho$, $c_p$ constant

radiation neglected

**Analysis:** (a)

$$\dot{E}_{in} - \dot{E}_{out,1} - \dot{E}_{out,2} + \dot{E}_g = \dot{E}_{st}$$

$$\boxed{\dot{E}_{in} = A\,q'' = -\pi R^2 k \frac{dT(x)}{dx}} \quad \boxed{\dot{E}_{out,1} = -\pi R^2 k \frac{dT(x+dx)}{dx}} \quad \boxed{\dot{E}_{out,2} = (2\pi R \cdot dx)\,h\,(T(x) - T_\infty)}$$

$$\boxed{\dot{E}_g = \dot{q} \cdot \pi R^2 dx} \quad \boxed{\dot{E}_{st} = m c_p \frac{dT}{dt} \cdot \pi R^2 dx}$$

$$m c_p \frac{dT(x)}{dt} \cdot \pi R^2 dx = (-\pi R^2 k)\left[\frac{dT(x)}{dx} - \frac{dT(x+dx)}{dx}\right] - (2\pi R h \cdot dx)(T(x) - T_\infty) + \dot{q} \cdot \pi R^2 dx$$

(b)

$$m c_p \left(\frac{dT(x)}{dt}\right) \cancel{\pi} R^{\cancel{2}} \cancel{dx} = (\cancel{\pi} R^2 k)\,\cancel{dx}\left(\frac{d^2T(x)}{dx^2}\right) - 2\cancel{\pi} R h \cdot \cancel{dx}\,(T(x) - T_\infty) + \dot{q} \cdot \cancel{\pi} R^{\cancel{2}}\,\cancel{dx}$$

$$\boxed{m \cdot c_p \cdot R \left(\frac{dT(x)}{dt}\right) = R \cdot k \left(\frac{d^2T(x)}{dx^2}\right) - 2h\,(T(x)) + (2hT_\infty + \dot{q})}$$

(c)

$$\boxed{T(0,t) = T_1} \qquad \boxed{T(x,0) = T_i}$$

$$\boxed{\left.\frac{dT}{dx}\right|_{x=L} = \frac{h}{-k}\left(T(L,t) - T_\infty\right)}$$

**#2** Known: Wall A: $L_A = .3\,m$, $\dot{q}$, $k_A$, insulated on the left side

Wall B: $L_B = .1\,m$, $k_B = \frac{30\,W}{m \cdot K}$

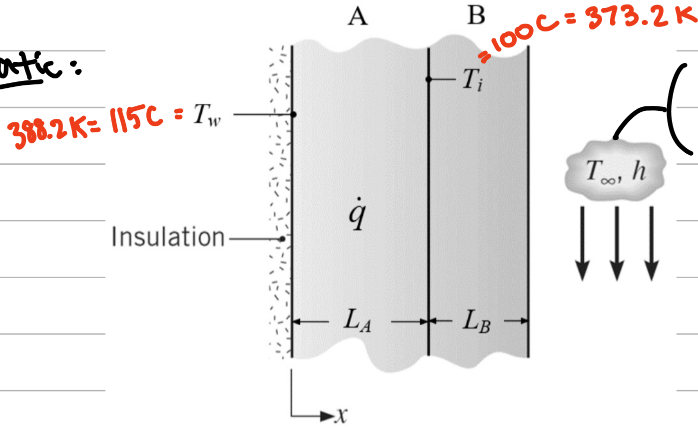
**Find:** (a) Right surface temp of wall B, $T_s$

(b) $\dot{q}$ @ steady state

(c) $k_A$ of wall A

(d) Plot $T(x)$ over both walls & show important features

**Schematic:**




$T_\infty = 20°C = 293.15$

$h = 500 \frac{W}{m^2 \cdot K}$

**Assumptions:** No radiation, can treat as 1D

**Analysis:** (a) @ end of wall B: $\dot{q}_{cond} \rightarrow \Big| \rightarrow \dot{q}_{conv}$ $\Big\}$ $\dot{q}_{cond} - \dot{q}_{conv} = 0$ ($T_s$)

$$A \cdot k \frac{\Delta T}{L_B} - Ah(T_s - T_\infty) = A\left[30 \cdot \frac{(373.15 - T_B)}{.1} - 500\,(T_B - 293.15)\right] = 0$$

$$300(373.15 - T_B) - 500\,(T_B - 293.15) = 0$$

solve with Ti-84 graphing: $\boxed{T_B = 323.15}$

(b) Analyzing between A & B: $q''_{in} \rightarrow \Big| \rightarrow q''_{out}$ , $T_i = 373.15\,K$

$$q''_{in} = \dot{q} \cdot \frac{A \cdot L}{A} \qquad q''_{out} = 30\,\frac{(373.15 - 323.15)}{.1}$$

$$q''_{in} - q''_{out} = 0$$

$$\dot{q} \cdot L - 300\,(373.15 - 323.15) = 0$$

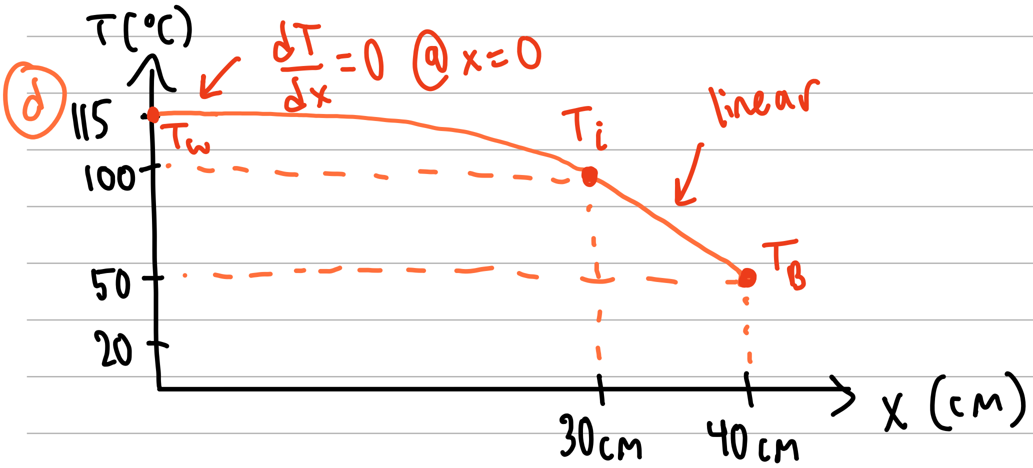
$$\boxed{\dot{q} = 300(373.15 - 323.15) \cdot \frac{1}{.3} = 50000 \frac{W}{m^3}}$$

(c) $\frac{d^2T}{dx^2} = \frac{-\dot{q}}{k_A} \rightarrow \frac{dT}{dx} = \frac{-\dot{q}}{k_A}x + C \rightarrow$ @ $x = 0$, $\frac{dT}{dx} = 0$ (insulated)

so $\frac{dT}{dx} = \frac{-\dot{q}}{k_A}x \rightarrow T(x) = \frac{-\dot{q}x^2}{2k_A} + C \rightarrow T(0) = 388.2 \rightarrow C = 388.2$

$$T(x) = \frac{-\dot{q}x^2}{2k_A} + 388.2 \rightarrow T(L_A) = 373.2 = \frac{(-50000)(.3)^2}{2 \cdot k_A} + 388.2$$

Solve to get $\boxed{k_A = 150}$

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**#3** Known: $k_b = 25\ \frac{W}{m \cdot K}$ $k_{TBC} = 1\ \frac{W}{m \cdot K}$ $R''_{BA} = 10^{-4}\ \frac{m^2 K}{W}$

$T_{\infty,o} = 1616\ K$ $T_{\infty,i} = 400\ K$ $h_o = 1000$ $h_i = 500$

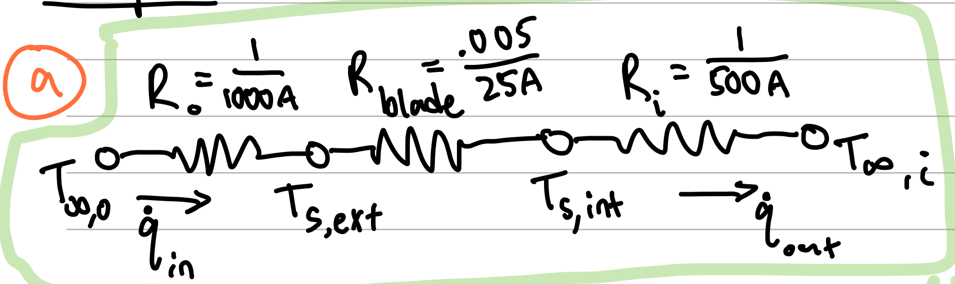
$L_b = .005\ m$ $T_{max} = 1200\ K$

Find: (a) if TBC not applied → draw thermal resistance network & label thermal resistances
↳ find $T_{s,ext}$, is it $< T_{max}$
(b) if TBC applied $L_{TBC} = 5mm$ → draw thermal resistance network & label thermal resistances
↳ find $T_{s,ext}$, is it $< T_{max}$

Assumptions: no radiation, 1D, Bonding negl. thickness

Analysis:

a




← $R_{conv} = \frac{1}{hA}$

← $R_{cond} = \frac{L}{kA}$

$$R_{total} = \frac{1}{1000A} + \frac{0.005}{25A} + \frac{1}{500A} \rightarrow \boxed{R''_{tot} = 0.0032}$$

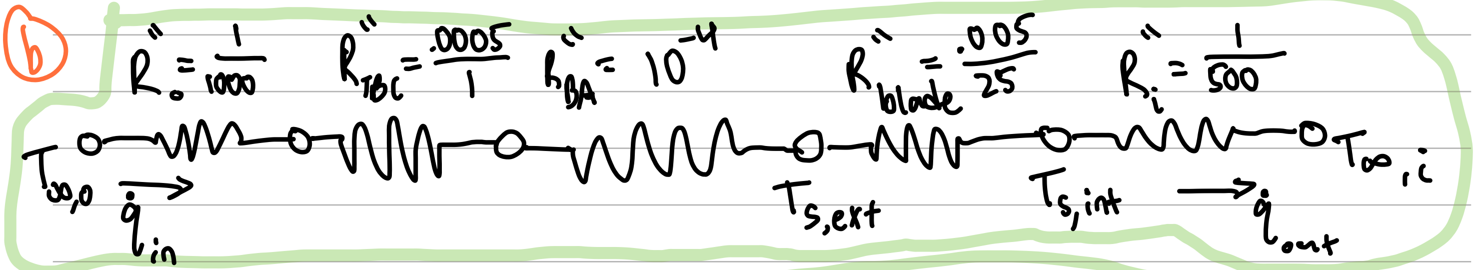
$$R''_{tot} = \frac{\Delta T_{tot}}{q''} = 0.0032 = \frac{1216}{q''} \rightarrow q'' = 380000$$

since it's in series all $q''$ are equal $\rightarrow R''_o = \frac{1}{1000} = \frac{(T_{\infty,o} - T_{s,ext})}{q''}$

$$\frac{1}{1000} = \frac{(1616 - T_{s,ext})}{380000} \rightarrow \boxed{T_{s,ext} = 1236\ K}$$

This <u>IS</u> more than $T_{max} = 1200\ K$

(b)




$$\boxed{R''_{tot} = \frac{1}{1000} + .0005 + 10^{-4} + \frac{.005}{25} + \frac{1}{500} = 0.0038}$$

$$R''_{tot} = \frac{\Delta T_{tot}}{q''} = 0.0038 = \frac{1216}{q''} \rightarrow q'' = 320000$$

$$R''_o + R''_{TBC} + R''_{BA} = \frac{1}{1000} + .0005 + 10^{-4} = \frac{(T_{\infty,o} - T_{s,ext})}{q''}$$

$$.0016 = \frac{(1616 - T_{s,ext})}{320000} \rightarrow \boxed{T_{s,ext} = 1104\ K}$$

This is <u>LESS</u> than $T_{max} = 1200\ K$